Formalising Approximation Fixpoint Theory

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Approximation Fixpoint Theory (AFT) is an abstract lattice-theoretic framework originally designed to unify semantics of non-monotonic logics [1]. Its first applications were on unifying all major semantics of logic programming, autoepistemic logic (AEL), and default logic (DL), thereby resolving a long-standing issue about the relationship between AEL and DL [2, 3]. AFT builds on Tarski's fixpoint theory of monotone operators on a complete lattice, starting from the key realisation that, by moving from the original lattice L to the bilattice L^2, Tarski's theory can be generalized into a fixpoint theory for arbitrary (i.e., also non-monotone) operators. Crucially, all that is required to apply AFT to a formalism and obtain several semantics is to define an appropriate approximating operator $L^2 \to L^2$ on this bilattice; the algebraic theory of AFT then directly defines different types of fixpoints that correspond to different types of semantics of the application domain.

In the last decade, AFT has seen several new application domains, including abstract argumentation, extensions of logic programming, extensions of autoepistemic logic, and active integrity constraints. Around the same time, also the theory of AFT has been extended significantly with new types of fixpoints, and results on *stratification*, *predicate introduction*, and *strong equivalence*. All of these results were developed in the highly general setting of lattice theory, making them directly applicable to all application domains, and such ensuring that researchers do not "reinvent the wheel".

Given the success and wide range of applicability of AFT, it sounded natural to formalise this theory in the Coq theorem prover. In this work we give a short introduction to AFT and the challenges of this formalisation.

References

[1] M. Denecker, V. Marek, and M. Truszczyński. Approximations, stable operators, well-founded fixpoints and applications in nonmonotonic reasoning. In J. Minker, editor, *Logic-Based Artificial Intelligence*, volume 597 of *The Springer International Series in Engineering and Computer Science*, pages 127–144. Springer US, 2000.

[2] M. Denecker, V. Marek, and M. Truszczyński. Uniform semantic treatment of default and autoepistemic logics. *Artif. Intell.*, 143(1):79–122, 2003.

[3] M. Denecker, V. Marek, and M. Truszczyński. Reiter's default logic is a logic of autoepistemic reasoning and a good one, too. In G. Brewka, V. Marek, and M. Truszczyński, editors, *Nonmonotonic Reasoning – Essays Celebrating Its 30th Anniversary*, pages 111–144. College Publications, 2011.

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