Causal reasoning in a logic with possible causal process semantics

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1 Theme of 2 talks

2 Motivation for this work

3 The logic

4 Actual causation

5 Conclusion
1 Outline

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1 A common theme of two talks: Construction Knowledge

- Construction knowledge
  - In many domains, certain objects can be constructed in terms of other objects
  - Human experts know how

- Two areas
  - Causal knowledge: the archetypical form?
    - the causal process
  - Inductive, recursive and other constructions in mathematics and in logics
    - the induction process
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2 Formalizing actual causation

What does it mean to say:

C caused E?
2 The counterfactual approach

- Lewis (1973):
  \[ C \text{ caused } E := \text{"Without C, E would not have been"} \]

- Pearl (2000)

- Halpern & Pearl (2005) (HP)


- Counterfactual definitions of actual causation in the context of structural equation models.
2 Criticisms against counterfactual definitions

- Objections against counterfactual approach
  Regulatory definitions

- Problematic causal scenarios for all counterfactual definitions
  ⇒ Refinements of the definitions
Halpern (2016b) analyzes 6 of these problematic causal scenarios.
2 Analysis

Halpern (2016b), scenario A

"There are four endogenous binary variables, A, B, C, and S, taking values 1 (on) and 0 (off). Intuitively, A and B are supposed to be alternative causes of C, and S acts as a switch. If S = 0, the causal route from A to C is active and that from B to C is dead; and if S = 1, the causal route from A to C is dead and the one from B to C is active."
2 Analysis

Halpern (2016b), scenario A

"There are four endogenous binary variables, A, B, C, and S, taking values 1 (on) and 0 (off). Intuitively, A and B are supposed to be alternative causes of C, and S acts as a switch. If S = 0, the causal route from A to C is active and that from B to C is dead; and if S = 1, the causal route from A to C is dead and the one from B to C is active."

\[ C := (\neg S \land A) \lor (S \land B) \]
2 Analysis

Halpern (2016b), scenario A

"There are four endogenous binary variables, A, B, C, and S, taking values 1 (on) and 0 (off). Intuitively, A and B are supposed to be alternative causes of C, and S acts as a switch. If \( S = 0 \), the causal route from A to C is active and that from B to C is dead; and if \( S = 1 \), the causal route from A to C is dead and the one from B to C is active."

\[
C := (\neg S \land A) \lor (S \land B)
\]

What is the actual cause of \( C \), intuitively?

- when \( S \), then \( A \)
- when \( \neg S \), then \( B \)
Analysis

Halpern (2016b), scenario A’

"But now consider a slightly different story. This time, we view B as the switch, rather than S. If B = 1, then C = 1 if either A = 1 or S = 1; if B = 0, then C = 1 only if A = 1 and S = 0."

\[
C = (B \land (A \lor S)) \lor (\neg B \land (A \land \neg S))
\]

What is the actual cause of \(C\), intuitively?

- when \(B\), then \(A\) or \(S\) or both
- when \(\neg B\), then \(A\) and \(\neg S\)
2 Analysis

Halpern (2016b), scenario A’

"But now consider a slightly different story. This time, we view B as the switch, rather than S. If \( B = 1 \), then \( C = 1 \) if either \( A = 1 \) or \( S = 1 \); if \( B = 0 \), then \( C = 1 \) only if \( A = 1 \) and \( S = 0 \)."

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\[ C := (B \land (A \lor S)) \lor (\neg B \land (A \land \neg S)) \]

What is the actual cause of C, intuitively?

- when B, then A or S or both
- when \( \neg B \), then A and \( \neg S \)
Structural equation models are ambiguous

- The two structural equations are mathematically equivalent!
- In both scenarios, the possible causal worlds are the same.
- Yet, the intuitive answer to actual causation problems is different.
- HP is correct in one of the scenarios.
2 Structural equation models are ambiguous

- The two structural equations are mathematically equivalent!
- In both scenarios, the possible causal worlds are the same.
- Yet, the intuitive answer to actual causation problems is different.
- HP is correct in one of the scenarios.

These structural equation models are ambiguous.
2 Structural equation models are ambiguous

- It must be the case that some information of these informal scenarios is not expressed by the structural equation model.
- This information does not affect the possible causal worlds.
- This information affects the answer to actual causation problems!
Structural equation models are ambiguous

- It must be the case that some information of these informal scenarios is not expressed by the structural equation model.
- This information does not affect the possible causal worlds.
- This information affects the answer to actual causation problems!

What kind of information is that? Let’s go back to the example.
2 The extra information

"There are four endogenous binary variables, A, B, C, and S, taking values 1 (on) and 0 (off). Intuitively, A and B are supposed to be alternative causes of C, and S acts as a switch. If S = 0, the causal route from A to C is active and that from B to C is dead; . . . ”

The extra information:

- separate causal mechanisms
- causes versus switches for causal mechanisms
- causal processes
- causal mechanisms can be alive or dead
  dead $\sim$ preempted
2 Solutions for the ambiguity

- Halpern’s solution is a KR methodology:
  “to add [...] extra variables, which [...] capture the mechanism of causality”.

- The approach of this paper:
  - Develop a formal language in which the missing information can be expressed.
  - Regulatory definition(s) of actual causation that exploits the extra information.
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We see separate causal mechanisms

Some sets of conditions trigger the causal mechanism

Other conditions could preempt the causal mechanism if not true.

Information about this strongly influences our idea of actual causation.
3 Syntax: Causal theories

Definition
A causal theory is a set of causal mechanisms.

Definition
A causal mechanism, or causal law, is an expression of the form

\[ A \leftarrow T \parallel P \]

where \( A \) is a literal, \( T \) and \( S \) sequences of literals

- A literal of \( T \) is called a trigger condition of the causal mechanism.
- A literal of \( P \) is called a no-preemption condition of the causal mechanism.
3 Example

- Scenario A:

\[
\begin{align*}
C & \leftarrow A \parallel \neg S \\
C & \leftarrow B \parallel S
\end{align*}
\]

- Scenario A’:

\[
\begin{align*}
C & \leftarrow A \parallel B \\
C & \leftarrow S \parallel B \\
C & \leftarrow A, S \parallel \neg B
\end{align*}
\]

- We made the information explicit that was available in Halperns informal domain description.
A possible causal world semantics is not refined enough.
3 Semantics: possible causal processes

The formal semantics specifies, for a causal theory $\Delta$:
- the possible causal processes of $\Delta$
- the possible causal world that each process leads to.

How to formalize the causal process?
- a causal process $\sim$ a dependency graph of the causal mechanisms that fire.
3 Another example: double preemption

Hall (2004)

Suzy fires a missile (SuzyF) to bomb a target (B); Enemy fires a missile (EnemyF) to hit Suzy’s missile (SuzyMH) and Billy fires a missile (BillyF) to hit Enemy’s missile (EnemyMH).

Theory:

\[
\begin{align*}
B & \leftarrow SuzyF \parallel \neg SuzyMH \\
SuzyMH & \leftarrow EnemyF \parallel \neg EnemyMH \\
EnemyMH & \leftarrow BillyF \\
\end{align*}
\]
3 Another example: double preemption

\[
\begin{align*}
  B & \leftarrow SuzyF \parallel \neg SuzyMH \\
  SuzyMH & \leftarrow EnemyF \parallel \neg EnemyMH \\
  EnemyMH & \leftarrow BillyF
\end{align*}
\]
3 Another example: double preemption

\[
\begin{align*}
B & \leftarrow \text{SuzyF} \parallel \neg \text{SuzyMH} \\
\text{SuzyMH} & \leftarrow \text{EnemyF} \parallel \neg \text{EnemyMH} \\
\text{EnemyMH} & \leftarrow \text{BillyF} \parallel
\end{align*}
\]

\[
\begin{array}{c}
\text{B} \\
\text{SuzyF} \\
\quad \downarrow \\
\text{EnemyF} \\
\quad \downarrow \\
\text{BillyF}
\end{array}
\quad
\begin{array}{c}
\neg \text{B} \\
\text{SuzyF} \\
\quad \uparrow \\
\text{EnemyF} \\
\quad \uparrow \\
\neg \text{BillyF}
\end{array}
\]

Causal reasoning
3 Derived concepts and properties

Derived concepts:

▶ An actual possible causal process induces a unique possible causal world
  • The possible causal process semantics is more refined than the possible world semantics.

▶ In a possible world, a causal mechanism can be:
  • firing
  • triggered but preempted
  • non-triggered
3 Derived concepts and properties

Derived concepts:

- An actual possible causal process induces a unique possible causal world
  - The possible causal process semantics is more refined than the possible world semantics.

- In a possible world, a causal mechanism can be:
  - firing
  - triggered but preempted
  - non-triggered

Some derived properties:

- All processes in the same exogeneous state cause the same possible world (confluence property)

- The possible causal worlds of $\Delta$ are the causal worlds of the structural equation model $Completion(\Delta)$
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4 Definitions of actual causation

The possible causal process is a detailed explanation of the world.

$x$ is an influence of $y$ in possible causal process $\mathcal{P}$

$x$ is an actual $\mathcal{P}$-cause of $y$

$x$ is an actual $\mathcal{DP}$-cause of $y$
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5 Conclusions

- A study of several sorts of knowledge that are important for actual causation but are not or not well expressed in many causal languages.
- Logic equipped with a possible causal process semantics.
- Some fundamental aspects of causation: the confluence of causal processes and, paradoxically, a theorem explaining why many useful causation problems can be solved without modelling mechanisms and processes.
- A rich and flexible framework for defining several notions of actual causation.
5 Future work

- Counterfactual definitions versus Regularity definition: two different sides of the same coin?
- Extending the logic: predicate logic, cyclic causal theories, ...
5 Implementation on-line


▶ An on-line implementation of many of the examples in the paper
▶ Using the knowledge base system IDP