On Local Domain Symmetry for Model Expansion

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Intro

General symmetry definition

Given a vocabulary Σ, theory T, and domain D, a symmetry σ for T is a permutation on the set of D,Σ-structures Γ_D such that for all $I \in \Gamma_D$:

$$I \models T \text{ iff } \sigma(I) \models T$$

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Why study symmetry?
speeding up search – symmetry breaking
avoid parts of the search space symmetrical to failed parts

Outline

Prelims: First-Order Logic (FO)

- vocabulary Σ of (function and predicate) symbols S/k
- Σ-theory T
- Σ-structure I
  - domain D
  - interpretations S^I for all $S \in \Sigma$

Semantics captured by satisfiability relation:

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In ASP: program $\leftrightarrow$ theory, set of facts $\leftrightarrow$ structure.
For the rest of the talk: vocabulary and domain are implicit and fixed.

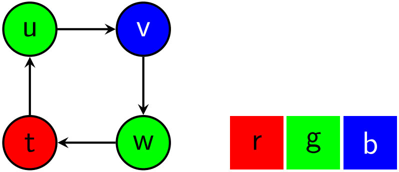
Running example: graph coloring

T_{gc}:

$$\forall x_1\ y_1 : Edge(x_1, y_1) \Rightarrow Color(x_1) \neq Color(y_1)$$
$$\forall x_2\ y_2 : Edge(x_2, y_2) \Rightarrow V(x_2) \wedge V(y_2)$$
$$\forall x_3 : C(Color(x_3))$$

I_{gc}:

$$V^{I_{gc}} = \{t, u, v, w\} \quad C^{I_{gc}} = \{r, g, b\}$$
$$Edge^{I_{gc}} = \{(t, u), (u, v), (v, w), (w, t)\}$$
$$Color^{I_{gc}} = t \mapsto r, u \mapsto g, v \mapsto b, w \mapsto g, r \mapsto r, g \mapsto g, b \mapsto b$$



Note: $I_{gc} \models T_{gc}$

Symmetry for a theory

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Symmetries compose to symmetry groups

Global Domain Symmetry

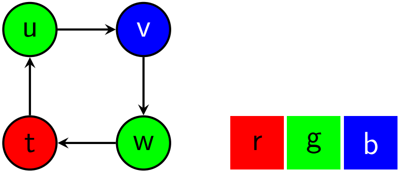
Permutation π on D induces permutation σ_π on Γ_D:

$$(\pi(d_1),\ldots,\pi(d_n)) \in P^{\sigma_\pi(I)} \text{ iff } (d_1,\ldots,d_n) \in P^I$$
$$f^{\sigma_\pi(I)}(\pi(d_1),\ldots,\pi(d_n)) = \pi(d_0) \text{ iff } f^I(d_1,\ldots,d_n) = d_0$$

Let's call such induced σ_π a Global Domain Symmetry for T.
Intuitively, domain renaming π preserves satisfiability:
$\sigma_\pi(I) \models T$ iff $I \models T$

Graph coloring ctd.

Let $\pi = (v\ r)$, so σ_π maps



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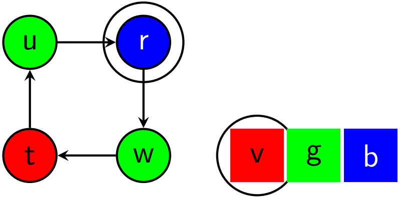

which still models T_{gc}.

Connectively closed argument positions

More precise notion of domain symmetry:
apply π only on limited set of argument positions A.

- Argument position $S|n$ with $S/k \in \Sigma$ and $1 \leq n \leq k$ denotes S's n^{th} argument. $f|0$ is output argument position.

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- Argument positions are connected under theory T if one occurs as subterm of the other, if they are connected by $=$, or if they are connected by quantified variables.
- A set of argument positions A is connectively closed under T if no other argument positions of Σ are connected to A under T.

Intuitively, a partition of connectively closed argument positions under T corresponds to a well-defined typing of T.

Graph coloring ctd.

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Connectively closed argument position partition under T_{gc}:

- $\{V|1, Edge|1, Edge|2, Color|1\}$
- $\{C|1, Color|0\}$

Local Domain Symmetry

Permutation π on D and argument position set A induce permutation σ_π^A on Γ_D:

$$(\pi^A(d_1), \ldots, \pi^A(d_n)) \in P^{\sigma_\pi^A(I)} \text{ iff } (d_1, \ldots, d_n) \in P^I$$

$$f^{\sigma_\pi^A(I)}(\pi^A(d_1), \ldots, \pi^A(d_n)) = \pi^A(d_0) \text{ iff } f^I(d_1, \ldots, d_n) = d_0$$

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where π^A applies π only on argument positions in A.

If A is connectively closed under T, σ_π^A is a local domain symmetry of T.

Intuitively, σ_π^A permutes domain element tuples according to some type A of T.

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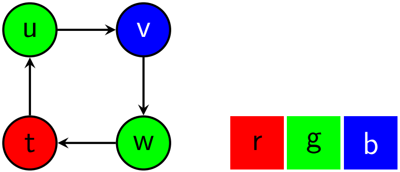

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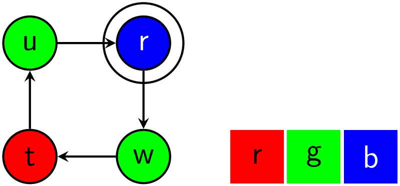

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- So far, so good: more or less known in literature (MACE, SEM, Paradox, ...)

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- What if we have to take pre-interpreted symbols into account? $\rightarrow$ Model eXpansion

First-Order Model Expansion (MX)

In:

- vocabulary $\Sigma = \Sigma_{in} \cup \Sigma_{out}$ $(\Sigma_{in} \cap \Sigma_{out} = \emptyset)$
- Σ-theory T
- Σ_{in}-structure I_{in}
    - domain D
    - interpretations $S^{I_{in}}$ to $S \in \Sigma_{in}$

Out:

- Σ_{out}-structure I_{out} such that $I_{in} \sqcup I_{out} \models T$
    - same domain D
    - $I_{in} \sqcup I_{out}$ merges both structures to a Σ-structure
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Shortened as $MX(T, I_{in})$.

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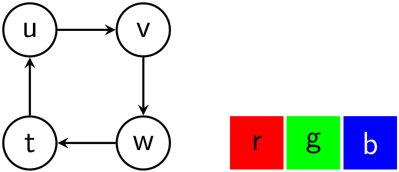

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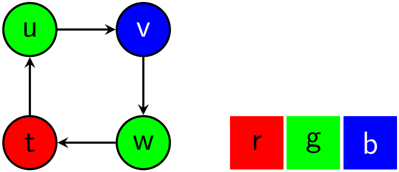
I_{gcout}:

$$Color^{I_{gcout}} = t \mapsto r, u \mapsto g, v \mapsto b, w \mapsto g, r \mapsto r, g \mapsto g, b \mapsto b$$

$I_{gcin} \sqcup I_{gcout}$:



Note: $I_{gcin} \sqcup I_{gcout} \models T_{gc}$

Symmetry for MX

Symmetry for $MX(T, I_{in})$

A symmetry σ for $MX(T, I_{in})$ is a permutation on the set of D,Σ_{out}-structures Γ_D such that for all $I \in \Gamma_D$:

$$I_{in} \sqcup I_{out} \models T \text{ iff } I_{in} \sqcup \sigma(I_{out}) \models T$$

Intuitively, symmetry must preserve input structure.

Sufficient condition for MX-symmetry?

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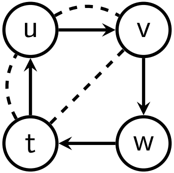
- Local domain symmetry σ_π^A where A is connectively closed under T and π such that $\sigma_\pi^A(I_{in}) = I_{in}$
- Does not work well for independent symbols connected under T:



Both graphs $Edge/2$ and $Edge'/2$ have different symmetries, but since their argument positions typically are connected by a $V/1$ argument position, no π exists that is consistent with the symmetry of both graphs.

Sufficient condition for MX-symmetry?

We defined transformation for $MX(T, I_{in})$ to $MX(T^*, I_{in}^*)$ such that

Sufficient condition for MX-symmetry

σ_π^A is a local domain symmetry for $MX(T, I_{in})$ if A is connectively closed under T^* and $\sigma_\pi^A(I_{in}^*) = I_{in}^*$.

Intuitively, $MX(T, I_{in})$ decouples all occurrences of pre-interpreted symbols.

Finding structure-preserving π?

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Answer:

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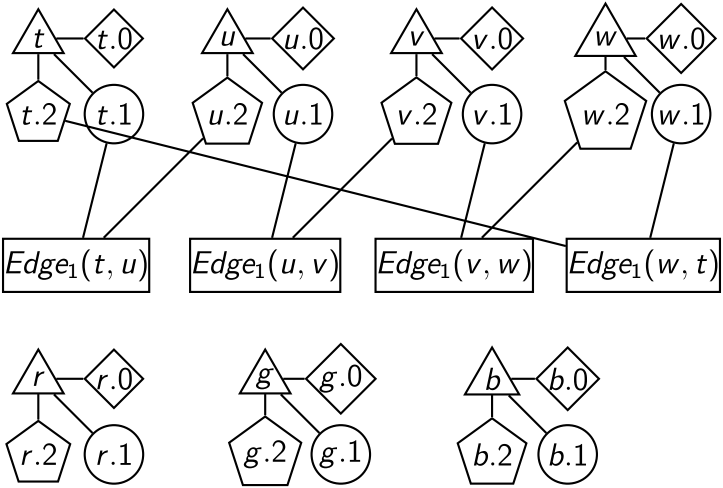
Answer:

- generate-and-test for domain element swaps $(d_1\ d_2)$
- encode to graph automorphism problem for more complicated π
  - In: I_{in}, A
  - Out: generators π that induce symmetry σ_π^A
  - Size of graph depends on size of I_{in}, not on size of T

Graph coloring ctd.



Symmetry breaking

Given symmetry group $\mathbb{G}$, construct symmetry breaking formula $\varphi(\mathbb{G})$.
$\varphi(\mathbb{G})$ is sound if for each I_{out}, there exists some $\sigma \in \mathbb{G}$ such that $I_{in} \sqcup \sigma(I_{out}) \models \varphi(\mathbb{G})$.
$\varphi(\mathbb{G})$ is complete if for each I_{out}, there exists exactly one $\sigma \in \mathbb{G}$ such that $I_{in} \sqcup \sigma(I_{out}) \models \varphi(\mathbb{G})$.

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What is the size of $\varphi(\mathbb{G})$ to break $\mathbb{G}$ completely?

Breaking local domain symmetry completely

$\mathbb{G}_\delta^A$ is a local domain interchangeability group if A is a set of argument positions, $\delta \subseteq D$, and each permutation π over δ induces a local domain symmetry σ_π^A.

$\mathbb{G}_\delta^A$ is broken completely with $O(|\delta|^2)$ sized symmetry breaking formula if A contains at most one argument position $S|i$ for each symbol $S \in \Sigma_{out}$.

Intuitively, after grounding, $\mathbb{G}_\delta^A$ represents row interchangeability of a Boolean variable matrix. Ordering the rows breaks all symmetry.

Graph coloring ctd.

Let $A = \{C|1, Color|0\}$, $\delta = \{r, b, g\}$. $\mathbb{G}_{\delta}^{A}$ represents the interchangeability of colors, and can be broken completely with a small symmetry breaking formula.

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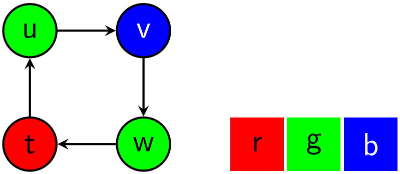
However, e.g. for the Ramsey number problem, $A = \{Edge|1, Edge|2, Color|1, \ldots\}$, so A' does not satisfy the "one argument position" condition.

Symmetry notions not captured by local domain symmetry

E.g. swapping colors of node v and t:



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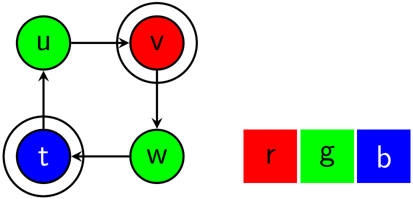

which still models T_{gc}.

Conclusion

- Notion of local domain symmetry
- Sufficient condition for symmetry detection in the context of an input structure
- Symmetry detection approach on predicate level
- Completeness guarantee for symmetry breaking
- Limits of our approach

- Notion can be extended to aggregates, non-monotonic rules, etc.
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Future work

Extend local domain symmetry to capture other notions of symmetry.

Thanks for your attention!

Questions?