Grounded fixpoints

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1 History & Motivation

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Operator-based semantics

Many logics: semantics defined as (some sort of) fixpoints of a semantic operator

- Logic programming
- Autoepistemic logic
- Default logic
- Dung’s argumentation frameworks
- Abstract dialectical frameworks
Operator-based semantics

- Many logics: semantics defined as (some sort of) fixpoints of a semantic operator
  - Logic programming
  - Autoepistemic logic
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- Lots of similarities, e.g.,
  - Monotone operator: least fixpoint
  - Non-monotone operator: harder to find “good fixpoints”
Approximation Fixpoint Theory

- Denecker, Marek and Truszczyński (2000) (DMT)
- Algebraical theory
- Defines different types of fixpoints of lattice operators
  - Supported fixpoints
  - (Partial) stable fixpoints
  - Kripke-Kleene fixpoint
  - Well-founded fixpoint
- Captures semantics of many logical formalisms
  - Logic programming
  - Autoepistemic logic
  - Default logic
  - Dung’s argumentation frameworks (Strass, 2013)
  - Abstract dialectical frameworks (Strass, 2013)
This paper

- Study phenomenon that occurs in the above domains
- Groundedness $\leftrightarrow$ self-supporting
Logic Programming

- Completion semantics (Clark, 1978)
Logic Programming

- Completion semantics (Clark, 1978)

\[ \{ p \leftarrow p \} : \text{self-supporting model } \{ p \} \]
Logic Programming

- Completion semantics (Clark, 1978)
- Perfect model semantics (Przymusinski, 1988)
- Well-founded semantics (Van Gelder et. al., 1988)
- Stable semantics (Gelfond and Lifschitz, 1988)
Autoepistemic Logic (AEL)

- Expansion semantics (Moore, 1985)
Autoepistemic Logic (AEL)

- Expansion semantics (Moore, 1985)

\{Kp \Rightarrow p\} : self-supporting model in which \(p\) is known
Autoepistemic Logic (AEL)

- Expansion semantics (Moore, 1985)
- “Honest theories” (Halpern and Moses, 1985)
- Moderately grounded expansions (Konolige, 1988)
- Strongly grounded expansions (Konolige, 1988)
- Stable semantics (DMT, 2000)
- Well-founded semantics (DMT, 2000)
Grounded fixpoints

- What is the problem with logic programs as \{p \leftarrow p\} or AEL theories such as \{Kp \Rightarrow p\}?
- Algebraical study (fixpoint theory)
- Application to logic programming, autoepistemic logic, default logic, Dung’s argumentation frameworks and abstract dialectical frameworks
Grounded fixpoints

- Complete lattice $\langle L, \leq \rangle$: every set $S \subseteq L$ has a least upper bound $\bigvee S$ and a greatest lower bound $\bigwedge S$
- Operator $O : L \to L$

**Definition (Grounded)**

We call $x \in L$ *grounded* for $O$ if for each $v \in L$ such that $O(x \land v) \leq v$, it holds that $x \leq v$. 
Intuition, if \( L = 2^F \), \( \leq = \subseteq \)

Then: \( \land = \cap \) and \( \lor = \cup \)

**Definition (Grounded)**

We call \( x \in L \) grounded for \( O \) if for each \( v \in L \) such that \( O(x \land v) \leq v \), it holds that \( x \leq v \).

\( x \) is grounded for \( O \) if it only contains facts that are sanctioned by \( O \): whenever we remove facts from \( x \), at least one of them is rederived.
Groundedness: Example

\[
\begin{align*}
&\{ p \leftarrow p. \\
&\{ q \leftarrow \neg p \lor q. \}
\end{align*}
\]

\[
\begin{array}{c}
\{ p \} \\
\{ q \} \\
\emptyset
\end{array}
\]
Properties of grounded fixpoints

**Proposition**

All grounded fixpoints of $O$ are minimal fixpoints of $O$.

**Proposition**

If $\mathcal{P}$ is a logic program, then grounded fixpoints of $T_{\mathcal{P}}$ are minimal supported models of $\mathcal{P}$.
Properties of grounded fixpoints

Proposition

A monotone operator has exactly one grounded fixpoint, namely its least fixpoint.

Proposition

If $\mathcal{P}$ is a positive logic program, then $T_\mathcal{P}$ has a unique grounded fixpoint, namely the least supported model of $\mathcal{P}$. 
Properties of grounded fixpoints

Proposition

All A-stable fixpoints of $O$ are grounded fixpoints of $O$.

Proposition

All stable models of a logic program $\mathcal{P}$ are grounded fixpoints of $T_\mathcal{P}$.
Properties of grounded fixpoints

**Proposition**

The well-founded fixpoint of a symmetric approximator $A$ of $O$ approximates all grounded fixpoints of $O$.

**Proposition**

The well-founded model of a logic program $\mathcal{P}$ is less precise than every grounded fixpoint of $T_\mathcal{P}$.
Properties of grounded fixpoints

**Proposition**

If the well-founded fixpoint of a symmetric approximator A of O is exact, then it is the unique grounded fixpoint of O.

**Proposition**

If the well-founded model of a logic program \( \mathcal{P} \) is two-valued, then it is the unique grounded fixpoint of \( T_\mathcal{P} \).
Grounded fixpoints in logic programming

- Study of existing semantics: stable and two-valued well-founded semantics are “grounded”
- Closely related to unfounded sets
- Grounded fixpoints induce a new semantics
  - Two-valued
  - Purely algebraical
  - Simple
  - Easily extensible
Grounded fixpoints in autoepistemic logic

- Superior formalisation of Konolige’s intuitions regarding “groundedness” (Konolige, 1988)
- The closest he got was “strong groundedness”, a syntactical criterion that depends on which rewriting to a normal form is used
- All strongly grounded expansions are *grounded fixpoints* and all *grounded fixpoints* are moderately grounded
Other applications of grounded fixpoints

- Default logic
- Dung’s argumentation frameworks
- Abstract dialectical frameworks
Results

- Abstract algebraical definition of groundedness
- Studied relation with other types of fixpoints from AFT
- Corresponds to intuitions in many different domains
- Study of existing semantics
- New induced semantics with attractive properties
- Studied complexity
Current and Future Work

- Grounded fixpoints (AAAI’15)
- Grounded fixpoints and their applications in knowledge representation (submitted)
- Partial grounded fixpoints (submitted)
  
  (Bogaerts, Vennekens and Denecker)